FIITJEE Solutions to

IIT - JEE – 2008

(Paper – 1, Code–7)

Time: 3 hours ***M. Marks: 246***

Note: ***(i)*** The question paper consists of 3 parts (Part I : Mathematics, Part II : Physics, Part III : Chemistry). Each part has 4 sections.

(ii) **Section I** contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct.

(iii) **Section II** contains 4 multiple correct answer type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **one or more answers** are correct.

(vi) **Section III** contains 4 Reasoning type questions. Each question contains STATEMENT–1 and STATEMENT–2.
Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1
Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1
Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.

(iv) **Section IV** contains 3 sets of Linked Comprehension type questions. Each set consists of a paragraph followed by 3 questions. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct.

Marking Scheme:

(i) For each question in **Section I**, you will be awarded **3 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

(ii) For each question in **Section II**, you will be awarded **4 Marks** if you have darkened all the bubble(s) corresponding to the correct answer and **zero mark** for all other cases. It may be noted that there is **no negative marking** for **wrong answer**.

(iii) For each question in **Section III**, you will be awarded **3 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

(iv) For each question in **Section IV**, you will be awarded **4 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

Mathematics

PART – I

SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents
(A) four straight lines, when c = 0 and a, b are of the same sign
(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a
(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a
(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

Sol. **(B)**

$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$

$\Rightarrow ax^2 + by^2 + c = 0$ or $x^2 - 5xy + 6y^2 = 0$

$\Rightarrow x^2 + y^2 = \left(-\frac{c}{a}\right)$ iff a = b, x – 2y = 0 and x – 3y = 0

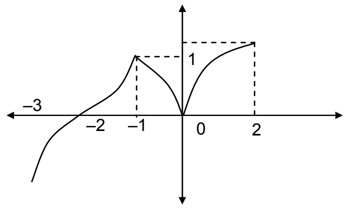
Hence the given equation represents two straight lines and a circle, when a = b and c is of sign opposite to that of a.

2. The total number of local maxima and local minima of the function $f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ is

(A) 0
(B) 1
(C) 2
(D) 3

Sol. **(C)**

Local maximum at x = – 1
and local minimum at x = 0
Hence total number of local maxima and local minima is 2.

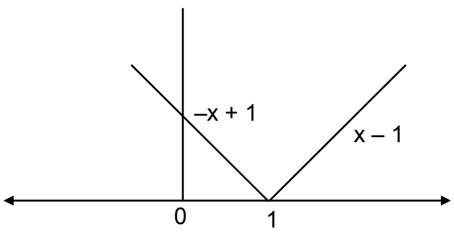


3. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}$; 0 < x < 2, m and n are integers, m ≠ 0, n > 0, and let p be the left hand derivative of |x – 1| at x = 1. If $\lim_{x\to 1^+} g(x) = p$, then

(A) n = 1, m = 1
(B) n = 1, m = – 1
(C) n = 2, m = 2
(D) n > 2, m = n

Sol. **(C)**

From graph, p = – 1



$\Rightarrow \lim_{x\to 1^+} g(x) = -1$

$\Rightarrow \lim_{h\to 0} g(1+h) = -1$

$\Rightarrow \lim_{h\to 0}\left(\frac{h^n}{\log\cos^m h}\right) = -1$

$\Rightarrow \lim_{h\to 0}\frac{n\cdot h^{n-1}}{m\cdot(-\tan h)} = -\left(\frac{n}{m}\right)\lim_{h\to 0}\left(\frac{h^{n-1}}{\tan h}\right) = -1$, which holds if n = m = 2.

4. If $0 < x < 1$, then $\sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right) + \sin\left(\cot^{-1}x\right)\right\}^2 - 1\right]^{1/2}$ is equal to

(A) $\frac{x}{\sqrt{1+x^2}}$
(B) x
(C) $x\sqrt{1+x^2}$
(D) $\sqrt{1+x^2}$

Sol. **(C)**

$$\sqrt{1+x^2}\left[\left(x\cos\cot^{-1}x + \sin\cot^{-1}x\right)^2 - 1\right]^{1/2}$$

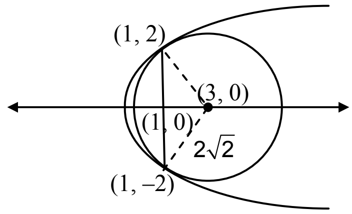
$$= \sqrt{1+x^2}\left[\left(x\cos\cos^{-1}\frac{x}{\sqrt{1+x^2}} + \sin\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}\left[\left(\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}\left(x^2+1-1\right)^{1/2} = x\sqrt{1+x^2}.$$

5. Consider the two curves $C_1 : y^2 = 4x$, $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then,
(A) C_1 and C_2 touch each other only at one point
(B) C_1 and C_2 touch each other exactly at two points
(C) C_1 and C_2 intersect (but do not touch) at exactly two points
(D) C_1 and C_2 neither intersect nor touch each other

Sol. **(B)**
The circle and the parabola touch each other at x = 1 i.e. at the points (1, 2) and (1, – 2) as shown in the figure.



6. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = 1/2$. Then the volume of the parallelopiped is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$

Sol. **(A)**

$$\text{Volume} = \left|\hat{a}\cdot\left(\hat{b}\times\hat{c}\right)\right| = \sqrt{\begin{vmatrix} \hat{a}\cdot\hat{a} & \hat{a}\cdot\hat{b} & \hat{a}\cdot\hat{c} \\ \hat{b}\cdot\hat{a} & \hat{b}\cdot\hat{b} & \hat{b}\cdot\hat{c} \\ \hat{c}\cdot\hat{a} & \hat{c}\cdot\hat{b} & \hat{c}\cdot\hat{c} \end{vmatrix}}$$

$$= \sqrt{\begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}} = \frac{1}{\sqrt{2}}.$$

SECTION – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

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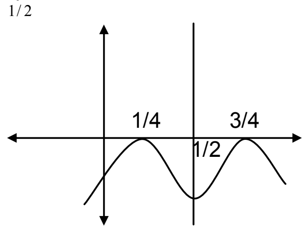
7. Let f (x) be a non-constant twice differentiable function defined on (– ∞, ∞) such that f (x) = f (1 – x) and $f'\left(\frac{1}{4}\right) = 0$. Then

(A) $f''(x)$ vanishes at least twice on [0, 1] (B) $f'\left(\frac{1}{2}\right) = 0$

(C) $\int_{-1/2}^{1/2} f\left(x + \frac{1}{2}\right)\sin x\,dx = 0$ (D) $\int_0^{1/2} f(t)e^{\sin \pi t}dt = \int_{1/2}^{1} f(1-t)e^{\sin \pi t}\,dt$

Sol. **(A, B, C, D)**
f (x) = f (1 – x)
Put x = 1/2 + x

$$f\left(\frac{1}{2} + x\right) = f\left(\frac{1}{2} - x\right)$$



Hence f (x + 1/2) is an even function or f (x + 1/2) sin x is an odd function.
Also, $f'(x) = -f'(1 - x)$ and for x = 1/2, we have $f'(1/2) = 0$.

Also, $\int_{1/2}^{1} f(1-t)e^{\sin \pi t}dt = -\int_{1/2}^{0} f(y)e^{\sin \pi y}dy$ (obtained by putting, 1 – t = y).

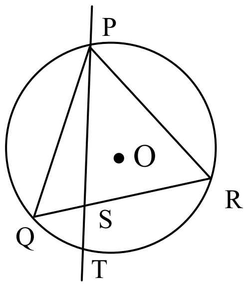
Since $f'(1/4) = 0$, $f'(3/4) = 0$. Also $f'(1/2) = 0$
⇒ $f''(x) = 0$ atleast twice in [0, 1] (Rolle's Theorem)

8. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$ (B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$ (D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

Sol. **(B, D)**



$PS \times ST = QS \times SR$

$\frac{\frac{1}{PS}+\frac{1}{ST}}{2} > \sqrt{\frac{1}{PS}\times\frac{1}{ST}}$

$\Rightarrow \frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

$\frac{QS+SR}{2} > \sqrt{QS\times SR}$

$\frac{QR}{2} > \sqrt{QS\times SR} \Rightarrow \frac{1}{\sqrt{QS\times SR}} > \frac{2}{QR}$

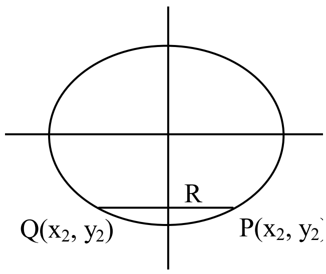
$\Rightarrow \frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$.

9. Let P (x_1, y_1) and Q (x_2, y_2), $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2+2\sqrt{3}\,y=3+\sqrt{3}$ (B) $x^2-2\sqrt{3}\,y=3+\sqrt{3}$

(C) $x^2+2\sqrt{3}\,y=3-\sqrt{3}$ (D) $x^2-2\sqrt{3}\,y=3-\sqrt{3}$

Sol. **(B, C)**



$\frac{x^2}{4}+\frac{y^2}{1}=1$

$b^2 = a^2(1-e^2)$

$\Rightarrow e = \frac{\sqrt{3}}{2}$

$\Rightarrow P\left(\sqrt{3}, -\frac{1}{2}\right)$ and $Q\left(-\sqrt{3}, -\frac{1}{2}\right)$ (given y_1 and y_2 less than 0).

Co-ordinates of mid-point of PQ are

$R \equiv \left(0, -\frac{1}{2}\right)$.

$PQ = 2\sqrt{3}$ = length of latus rectum.

$\Rightarrow$ two parabola are possible whose vertices are $\left(0, -\frac{\sqrt{3}}{2}-\frac{1}{2}\right)$ and $\left(0, \frac{\sqrt{3}}{2}-\frac{1}{2}\right)$.

Hence the equations of the parabolas are $x^2 - 2\sqrt{3}y = 3+\sqrt{3}$

and $x^2 + 2\sqrt{3}y = 3-\sqrt{3}$.

10. Let $S_n = \sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$ for n = 1, 2, 3, …. Then,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$ (B) $S_n > \frac{\pi}{3\sqrt{3}}$

(C) $T_n < \frac{\pi}{3\sqrt{3}}$ (D) $T_n > \frac{\pi}{3\sqrt{3}}$

Sol. **(A, D)**

$S_n < \lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{n}\cdot\frac{1}{1+k/n+(k/n)^2}$

$= \int\limits_0^1 \frac{dx}{1+x+x^2} = \frac{\pi}{3\sqrt{3}}$

Now, $T_n > \frac{\pi}{3\sqrt{3}}$ as $h\sum\limits_{k=0}^{n-1} f(kh) > \int\limits_0^1 f(x)dx > h\sum\limits_{k=1}^{n} f(kh)$

SECTION – III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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11. Consider the system of equations $ax + by = 0$, $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$.

STATEMENT – 1: The probability that the system of equations has a unique solution is 3/8.

and

STATEMENT – 2: The probability that the system of equations has a solution is 1.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(B)**

For unique solution $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$ where $a, b, c, d \in \{0, 1\}$

Total cases = 16.

Favorable cases = 6 (Either ad = 1, bc = 0 or ad = 0, bc = 1).

Probability that system of equations has unique solution is $\frac{6}{16} = \frac{3}{8}$ and system of equations has either unique solution or infinite solutions so that probability for system to have a solution is 1.

12. Consider the system of equations

$x - 2y + 3z = -1$
$-x + y - 2z = k$
$x - 3y + 4z = 1$

STATEMENT -1 : The system of equations has no solution for $k \neq 3$.

and

STATEMENT -2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(A)**

$$D = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$$

and $D_1 = \begin{vmatrix} -1 & -2 & 3 \\ k & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = (3-k) = 0$ if $k = 3$

$D_2 = \begin{vmatrix} 1 & -1 & 3 \\ -1 & k & -2 \\ 1 & 1 & 4 \end{vmatrix} = (k-3) = 0$, if $k = 3$

$D_3 = \begin{vmatrix} 1 & -2 & -1 \\ -1 & 1 & k \\ 1 & -3 & 1 \end{vmatrix} = (k-3) = 0$, if $k = 3$

$\Rightarrow$ system of equations has no solution for $k \neq 3$.

13. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is continuous, $g(0) \neq 0$, $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$.

STATEMENT -1 : $\lim\limits_{x \to 0}[g(x)\cot x - g(0)\text{cosec}x] = f''(0)$.

and

STATEMENT -2 : $f'(0) = g(0)$.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(B)**

$f'(x) = g(x)\cos x + \sin x . g'(x)$

$\Rightarrow f'(0) = g(0)$

$f''(x) = 2g'(x) \cos x - g(x) \sin x + \sin x\, g''(x)$

$\Rightarrow f''(0) = 2g'(0) = 0$

But $\lim\limits_{x \to 0}\left[g(x)\cot x - g(0)\text{cosec } x\right] = \lim\limits_{x \to 0} \dfrac{g(x)\cos x - g(0)}{\sin x} = \lim\limits_{x \to 0} \dfrac{g'(x)\cos x - g(x)\sin x}{\cos x} = g'(0) = 0 = f''(0)$.

14. Consider three planes

$P_1 : x - y + z = 1$

$P_2 : x + y - z = -1$

$P_3 : x - 3y + 3z = 2$.

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.

STATEMENT -1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT -2 : The three planes do not have a common point.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**

The direction cosines of each of the lines L_1, L_2, L_3 are proportional to $(0, 1, 1)$.

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 17

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.
If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$.
If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

15. If $f\left(-10\sqrt{2}\right) = 2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right) =$

(A) $\dfrac{4\sqrt{2}}{7^3 3^2}$ (B) $-\dfrac{4\sqrt{2}}{7^3 3^2}$
(C) $\dfrac{4\sqrt{2}}{7^3 3}$ (D) $-\dfrac{4\sqrt{2}}{7^3 3}$

Sol. **(B)**
Differentiating the given equation, we get
$3y^2y' - 3y' + 1 = 0$
$\Rightarrow y'\left(-10\sqrt{2}\right) = -\dfrac{1}{21}$
Differentiation again we get $6yy'^2 + 3y^2y'' - 3y'' = 0$
$\Rightarrow f''\left(-10\sqrt{2}\right) = -\dfrac{6.2\sqrt{2}}{(21)^4} = -\dfrac{4\sqrt{2}}{7^3 3^2}$.

16. The area of the region bounded by the curves $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is

(A) $\displaystyle\int_a^b \frac{x}{3\left(\left(f(x)\right)^2 - 1\right)}dx + bf(b) - af(a)$ (B) $\displaystyle-\int_a^b \frac{x}{3\left(\left(f(x)\right)^2 - 1\right)}dx + bf(b) - af(a)$

(C) $\displaystyle\int_a^b \frac{x}{3\left(\left(f(x)\right)^2 - 1\right)}dx - bf(b) + af(a)$ (D) $\displaystyle-\int_a^b \frac{x}{3\left(\left(f(x)\right)^2 - 1\right)}dx - bf(b) + af(a)$

Sol. **(A)**

The required area = $\displaystyle\int_a^b f(x)dx = xf(x)\Big|_a^b - \int_a^b xf'(x)dx$

$$= bf(b) - af(a) + \int_a^b \frac{x}{3\left[(f(x)^2 - 1)\right]}dx.$$

17. $\displaystyle\int_{-1}^{1} g'(x)dx =$

(A) $2g(-1)$ (B) 0
(C) $-2g(1)$ (D) $2g(1)$

Sol. **(D)**
We have $y' = \dfrac{1}{3\left(1 - \left(f(x)^2\right)\right)}$ which is even

Hence $\displaystyle\int_{-1}^{1} g'(x) = g(1) - g(-1) = 2g(1)$.

Paragraph for Question Nos. 18 to 20

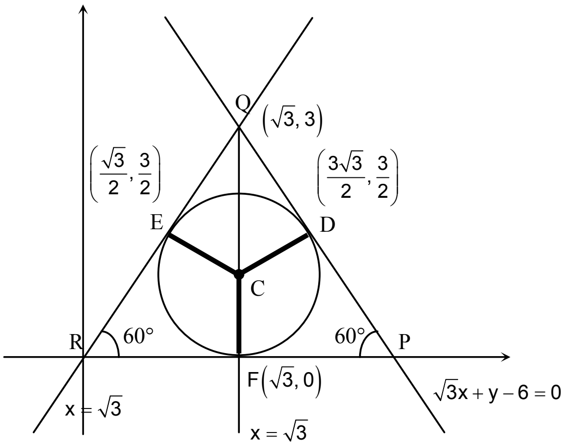
A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

18. The equation of circle C is

(A) $\left(x - 2\sqrt{3}\right)^2 + (y-1)^2 = 1$ (B) $\left(x - 2\sqrt{3}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(C) $\left(x - \sqrt{3}\right)^2 + (y+1)^2 = 1$ (D) $\left(x - \sqrt{3}\right)^2 + (y-1)^2 = 1$

Sol. **(D)**



Equation of CD is $\dfrac{x - \frac{3\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{y - \frac{3}{2}}{\frac{1}{2}} = -1$

$\Rightarrow C \equiv \left(\sqrt{3}, 1\right)$

Equation of the circle is $\left(x - \sqrt{3}\right)^2 + (y-1)^2 = 1$.

19. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$ (B) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\sqrt{3}, 0\right)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Sol. **(A)**

Since the radius of the circle is 1 and C $\left(\sqrt{3}, 1\right)$, coordinates of $F \equiv \left(\sqrt{3}, 0\right)$

Equation of CE is $\dfrac{x - \sqrt{3}}{-\frac{\sqrt{3}}{2}} = \dfrac{y-1}{\frac{1}{2}} = 1$

$\Rightarrow E \equiv \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$.

20. Equation of the sides QR, RP are

(A) $y = \frac{2}{\sqrt{3}}x + 1,\ y = -\frac{2}{\sqrt{3}}x - 1$ (B) $y = \frac{1}{\sqrt{3}}x,\ y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1,\ y = -\frac{\sqrt{3}}{2}x - 1$ (D) $y = \sqrt{3}x,\ y = 0$

Sol. **(D)**

Equation of QR is $y - 3 = \sqrt{3}\left(x - \sqrt{3}\right)$

$\Rightarrow\ y = \sqrt{3x}$

Equation of RP is y = 0.

Paragraph for Question Nos. 21 to 23

Let A, B, C be three sets of complex numbers as defined below

$A = \{z : \text{Im}z \ge 1\}$

$B = \left\{z : |z - 2 - i| = 3\right\}$

$C = \left\{z : \text{Re}\left((1 - i)z\right) = \sqrt{2}\right\}$.

21. The number of elements in the set $A \cap B \cap C$ is

(A) 0 (B) 1

(C) 2 (D) ∞

Sol. **(B)**

A = Set of points on and above the line y = 1 in the Argand plane.

B = Set of points on the circle $(x - 2)^2 + (y - 1)^2 = 3^2$

C = Re(1 – i) z = Re((1 – i) (x + iy))

$\Rightarrow x + y = \sqrt{2}$

Hence $(A \cap B \cap C)$ = has only one point of intersection.

22. Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

(A) 25 and 29 (B) 30 and 34

(C) 35 and 39 (D) 40 and 44

Sol. **(C)**

The points (– 1, 1) and (5, 1) are the extremities of a diameter of the given circle .

Hence $|z + 1 - i|^2 + |z - 5 - i|^2 = 36$.

23. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between

(A) –6 and 3 (B) –3 and 6

(C) –6 and 6 (D) –3 and 9

Sol. **(D)**

$\left||z| - |w|\right| < |z - w|$

and $|z - w|$ = Distance between z and w

z is fixed. Hence distance between z and w would be maximum for diametrically opposite points.

$\Rightarrow |z - w| < 6$

$\Rightarrow -6 < |z| - |w| < 6$

$\Rightarrow -3 < |z| - |w| + 3 < 9$.

Physics

PART - II

Useful Data:

Plank's constant h = 4.1×10^{-15} eV.s

Velocity of light c = 3×10^{8} m/s

SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

24. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be

(A) 30° for both the colours
(B) greater for the violet colour
(C) greater for the red colour
(D) equal but not 30° for both the colours

Sol. **(A)**

At minimum deviation for any wavelength

$r_1 = r_2 = A/2$, Because $r_1 + r_2 = A$

25. Which one of the following statements is **WRONG** in the context of X-rays generated from a X-ray tube?

(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases.
(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube
(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

Sol. **(B)**

$\lambda_{cutoff} = \frac{hc}{eV}$ (independent of atomic number)

26. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(A) $\frac{1}{T}$
(B) $\frac{2}{T}$
(C) $\frac{3}{T}$
(D) $\frac{4}{T}$

Sol. **(C)**

$\gamma = \frac{1}{V}\left(\frac{dV}{dT}\right)$

PT^2 = constant

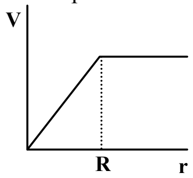
$\frac{nRT}{V}T^2$ = constant

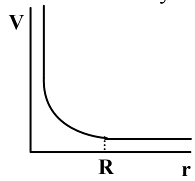
$\therefore \quad \gamma = \frac{3}{T}$

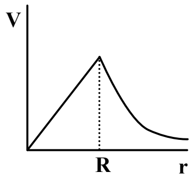
27. A spherically symmetric gravitational system of particles has a mass density

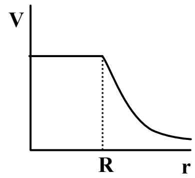
$$\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(A)


(B)


(C)


(D)


Sol. **(C)**

$G\frac{4}{3}\pi\rho_0 r = \frac{v^2}{r}$, $\quad r \le R$

Hence, $v \propto r$

$\left(\frac{G\frac{4}{3}\pi\rho_0 R^3}{r^2}\right) = \left(\frac{v^2}{r}\right)$, $\quad r \ge R$

Hence $\quad v \propto \frac{1}{\sqrt{r}}$

28. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and /or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	Length of the pendulum (cm)	Number of oscillations (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g, i.e., $\left(\frac{\Delta g}{g} \times 100\right)$ for students I, II and III, respectively,

(A) $E_I = 0$

(B) E_I is minimum

(C) $E_I = E_{II}$

(D E_{II} is maximum

Sol. **(B)**

$g = 4\pi^2\left(\frac{\ell}{T^2}\right)$

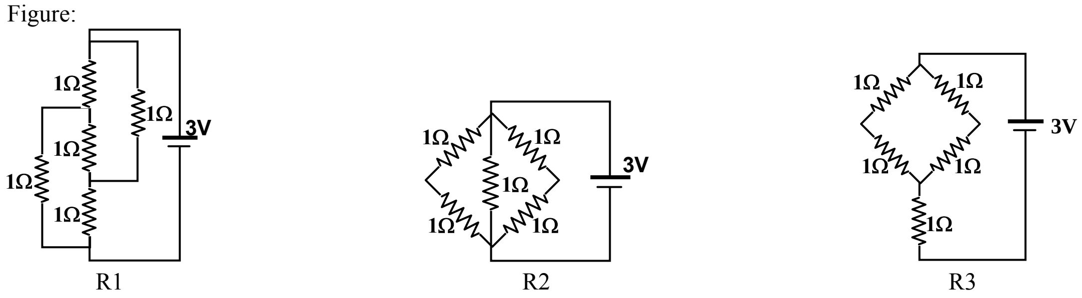
$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$

$\Rightarrow E = \frac{\Delta \ell}{\ell} + 2\frac{\Delta t}{t}$, greater the value of t, lesser the error

Hence, fractional error in the Ist observation is minimum

29. Figure shows three resistor configurations R1, R2 and R3 connected to 3V battery. If the power dissipated by the configuration R1, R2 and R3 is P1, P2 and P3, respectively, then

Figure:



(A) P1 > P2 >P3 (B) P1 > P3 >P2

(C) P2 > P1 >P3 (D) P3 >P2 > P1

Sol. **(C)**

$P = \frac{V^2}{R}$

$R_1 = 1\ \Omega$, $R_2 = 1/2\ \Omega$, $R_3 = 2\ \Omega$

$\therefore\ P_2 > P_1 > P_3$

SECTION – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

30. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).

(A) If $d = \lambda$, the screen will contain only one maximum

(B) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen

(C) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase

(D) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

Sol. **(A) & (B)**

For at least one maxima, $\sin\theta = \lambda/d$

If $\lambda = d$, $\sin\theta = 1$ and $y \to \infty$

If $\lambda < d < 2d$, $\sin\theta$ exists and y is finite

31. The balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option(s) is (are) **NOT ALLOWED** for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

(A) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$

(B) $\vec{p}_1' = c_1\hat{k}$, $\vec{p}_2' = c_2\hat{k}$

(C) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

Sol. **(A) & (D)**

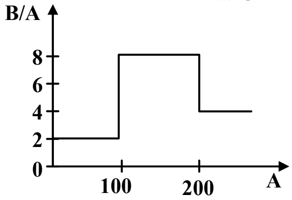
$\vec{P} = \vec{P}_1 + \vec{P}_2 = \vec{P}_1' + \vec{P}_2'$

$F_{ext} = 0$

$|\vec{P}| = 0$

32. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below.

Figure:

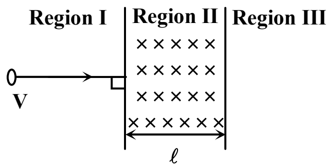


(A) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy
(B) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy
(C) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments
(D) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments

Sol. **(B) & (D)**
If $(BE)_{final} - (BE)_{initial} > 0$
Energy will be released.

33. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is ℓ. Choose the correct choice(s).

Figure:



(A) The particle enters Region III only if its velocity $V > \frac{q\ell B}{m}$

(B) The particle enters Region III only if its velocity $V < \frac{q\ell B}{m}$

(C) Path length of the particle in Region II is maximum when velocity $V = \frac{q\ell B}{m}$

(D) Time spent in Region II is same for any velocity V as long as the particle returns to Region I

Sol. **(A), (C) & (D)**

SECTION – III

Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

34. STATEMENT-1
The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

STATEMENT-2
In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(A)**

35. STATEMENT-1
Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

and

STATEMENT-2
By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(D)**
$a = \frac{mgR^2 \sin\theta}{I_{cm} + mR^2}$

36. STATEMENT-1
In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

and

STATEMENT-2
Resistance of a metal increases with increase in temperature.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(D)**
$R_{unknown} = \frac{R(100-\ell)}{\ell}$

37. STATEMENT-1
An astronaut in an orbiting space station above the Earth experiences weightlessness.

and

STATEMENT-2
An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'.
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT -1 is True, STATEMENT-2 is False
(D) STATEMENT -1 is False, STATEMENT-2 is True

Sol. **(A)**

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 38 to 40

In a mixture of H–He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

38. The quantum number n of the state finally populated in He^+ ions is
(A) 2 (B) 3 (C) 4 (D) 5

Sol. **(C)**

39. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is
(A) 6.5×10^{-7} m (B) 5.6×10^{-7} m
(C) 4.8×10^{-7} m (D) 4.0×10^{-7} m

Sol. **(C)**
$E_4 - E_3 = \dfrac{hc}{\lambda}$ [λ: visible region]

40. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

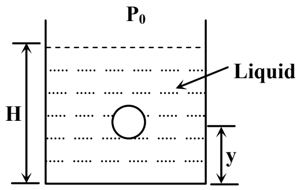
Sol. **(A)**
KE $\propto Z^2/n^2$

$$\frac{KE_H}{KE_{He}} = \left(\frac{Z_H}{2}\Big/\frac{Z_{He}}{2}\right)^2 = \frac{1}{4}$$

Paragraph for Question Nos. 41 to 43

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_ℓ (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension).

Figure:



41. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it
(A) Only the force of gravity
(B) The force due to gravity and the force due to the pressure of the liquid
(C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid
(D) The force due to gravity and the force due to viscosity of the liquid

Sol. **(D)**
Buoyant force is resultant of pressure-force of liquid.

42. When the gas bubble is at a height y from the bottom, its temperature is

(A) $T_0\left(\dfrac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{2/5}$ (B) $T_0\left(\dfrac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{2/5}$

(C) $T_0\left(\dfrac{P_0 + \rho_\ell gH}{P_0 + \rho_\ell gy}\right)^{3/5}$ (D) $T_0\left(\dfrac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell gH}\right)^{3/5}$

Sol. **(B)**

$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma}$

$P_1 = P_0 + \rho_\ell gH, \quad T_1 = T_0$

$P_2 = P_0 + \rho_\ell g(H - y)$

43. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $\rho_\ell nRgT_0 \dfrac{(P_0+\rho_\ell gH)^{2/5}}{(P_0+\rho_\ell gy)^{7/5}}$

(B) $\dfrac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{2/5}[P_0+\rho_\ell g(H-y)]^{3/5}}$

(C) $\rho_\ell nRgT_0 \dfrac{(P_0+\rho_\ell gH)^{3/5}}{(P_0+\rho_\ell gy)^{8/5}}$

(D) $\dfrac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{3/5}[P_0+\rho_\ell g(H-y)]^{2/5}}$

Sol. **(B)**

$\rho_\ell Vg = \text{Buoyancy force} = \rho_\ell g\dfrac{nRT_2}{P_2}$

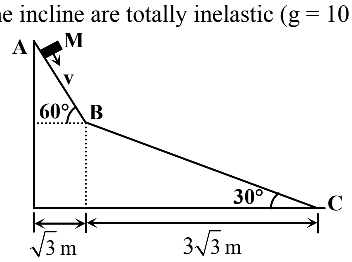
$T_2 = T_0\left[\dfrac{P_0+\rho_\ell g(H-y)}{P+\rho_\ell gH}\right]^{2/5}$

$P_2 = P_0 + \rho_\ell g(H - y)$

Paragraph for Question Nos. 44 to 46

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10\ m/s^2$).

Figure:



44. The speed of the block at point B immediately after it strikes the second incline is

(A) $\sqrt{60}$ m/s (B) $\sqrt{45}$ m/s (C) $\sqrt{30}$ m/s (D) $\sqrt{15}$ m/s

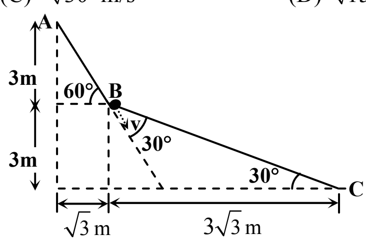
Sol. **(B)**

Along the plane velocity just before collision

$v = \sqrt{2g(3)} = \sqrt{60}$ m/s

Along the plane velocity just after collision

$v_B = v \cos 30° = \sqrt{45}$ m/s



45. The speed of the block at point C, immediately before it leaves the second incline is

(A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s (C) $\sqrt{90}$ m/s (D) $\sqrt{75}$ m/s

Sol. **(B)**

$mg(3) = \dfrac{1}{2}m\left(v_C^2 - v_B^2\right) \Rightarrow \quad v_C = \sqrt{105}$ m/s

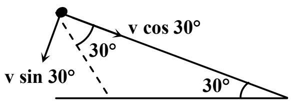
46. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

(A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s (C) 0 (D) $-\sqrt{15}$ m/s

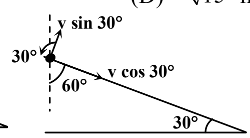
Sol. **(C)**

$v_y = v \sin 30° \cos 30° - v \cos 30° \cos 60°$

$v_y = 0$



Before collision



After collision

Chemistry

PART – III
SECTION – I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

47. Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the presence of
(A) nitrogen (B) oxygen
(C) carbon dioxide (D) argon

Sol. **(B)**
Ag dissociates in a solution of NaCN in the presence of air, and forms sodium argentocyanide.
$$4Ag + 8NaCN + 2H_2O + O_2 \longrightarrow 4Na\left[Ag(CN)_2\right] + 4NaOH$$

48. 2.5 mL of $\frac{2}{5}$M weak monoacidic base $\left(K_b = 1\times10^{-12} \text{ at } 25^0 C\right)$ is titrated with $\frac{2}{15}$M HCl in water at 25°C. The concentration of H^+ at equivalence point is $\left(K_w = 1\times10^{-14} \text{ at } 25^0 C\right)$
(A) 3.7×10^{-13} M (B) 3.2×10^{-7} M
(C) 3.2×10^{-2} M (D) 2.7×10^{-2} M

Sol. **(D)**
$$BOH + HCl \longrightarrow \underset{C}{BCl} + H_2O$$
$$\underset{C(1-h)}{B^+} + H_2O \rightleftharpoons \underset{Ch}{BOH} + \underset{Ch}{H^+}$$

Volume of HCl used $= \dfrac{2.5\times\frac{2}{5}}{2/15} = 7.5\,\text{ml}$

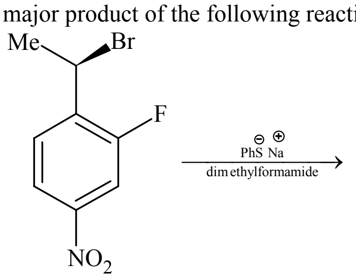
Concentration of Salt, C $= \dfrac{2.5\times\frac{2}{5}}{10} = 0.1\,\text{M}$

$$\therefore \frac{Ch^2}{1-h} = \frac{K_w}{K_b}$$
Solving h = 0.27
$$\left[H^+\right] = Ch = 0.1\times0.27 = 2.7\times10^{-2}\ \text{M}$$

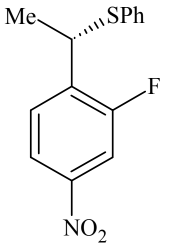
49. Under the same reaction conditions, initial concentration of 1.386 mol dm^{-3} of a substance becomes half in 40 seconds and 20 seconds through first order and zero order kinetics, respectively. Ratio $\left(\frac{k_1}{k_0}\right)$ of the rate constants for first order (k_1) and zero order (k_0) of the reactions is
(A) 0.5 mol^{-1} dm^3 (B) 1.0 mol dm^{-3}
(C) 1.5 mol dm^{-3} (D) 2.0 mol^{-1} dm^3

Sol. **(A)**
$$k_1 = \frac{0.693}{t_{1/2}} = \frac{0.693}{40}$$
$$k_0 = \frac{A_0}{2t_{1/2}} = \frac{1.386}{2\times20}$$
$$\frac{k_1}{k_0} = \frac{0.693}{40}\times\frac{40}{1.386} = \frac{0.693}{1.386} = 0.5\ \text{mol}^{-1}\text{litre}$$

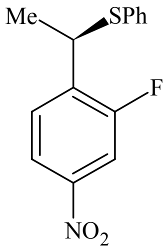
50. The major product of the following reaction is



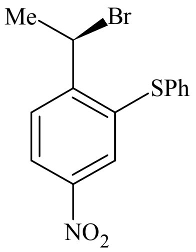
(A)



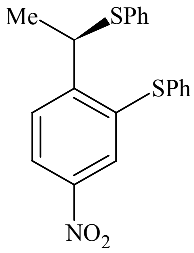
(B)



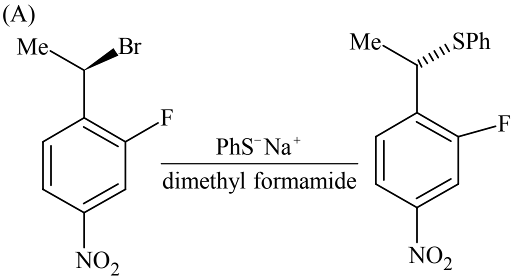
(C)



(D)



Sol. (A)



It is easier to do nucleophilic substitution on alkyl halides than on aryl halides.

51. Hyperconjugation involves overlap of the following orbitals

(A) σ - σ (B) σ - p
(C) p – p (D) π - π

Sol. **(B)**
Hyperconjugation involves overlap of $\sigma - p$ orbitals.

52. Aqueous solutions of $Na_2S_2O_3$ on reaction with Cl_2 gives

(A) $Na_2S_4O_6$ (B) $NaHSO_4$
(C) NaCl (D) NaOH

Sol. **(B)**
$$Na_2S_2O_3 + 4Cl_2 + 5H_2O \longrightarrow 2NaHSO_4 + 8HCl$$

SECTION – II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY OR MORE** is/are correct.

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53. A gas described by van der Waal's equation
(A) behaves similar to an ideal gas in the limit of large molar volumes
(B) behaves similar to an ideal gas in the limit of large pressures
(C) is characterized by van der Waal's coefficients that are dependent on the identify of the gas but are independent of the temperature
(D) has the pressure that is lower than the pressure exerted by the same gas behaving ideally

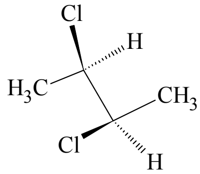
Sol. **(A, C, D)**

$$\left(P+\frac{n^2a}{V^2}\right)\left(V-nb\right)=nRT$$

At low pressure, when the sample occupies a large volume, the molecules are so far apart for most of the time that the intermolecular forces play no significant role, and the gas behaves virtually perfectly.

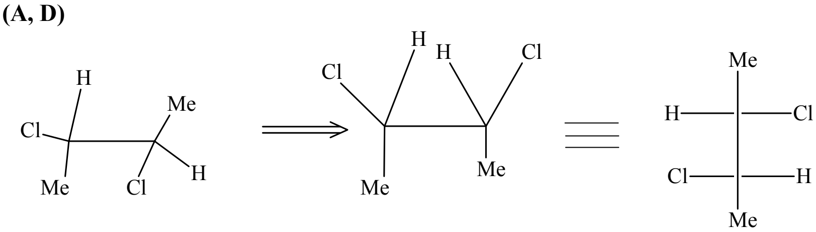
a and b are characteristic of a gas and are independent of temperature. The term $\left(P+\frac{n^2a}{V^2}\right)$ represents the pressure exerted by an ideal gas while P represents the pressure exerted by a real gas.

54. The correct statement (s) about the compound given below is (are)

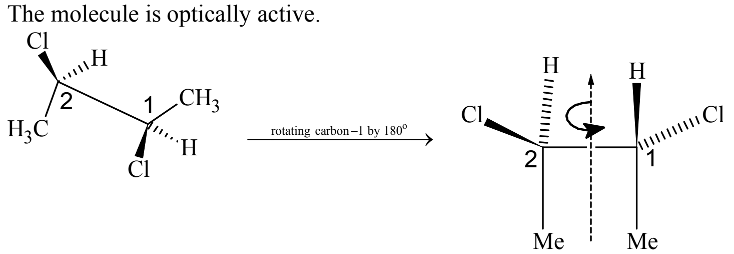


(A) The compound is optically active
(B) The compound possesses centre of symmetry
(C) The compound possesses plane of symmetry
(D) The compound possesses axis of symmetry

Sol. **(A, D)**

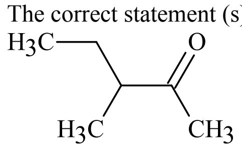


The molecule is optically active.

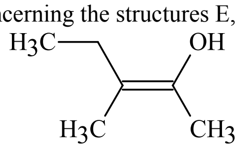


The molecule possesses an axis of symmetry (C_2) perpendicular to the C – C bond.

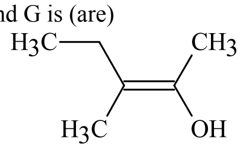
55. The correct statement (s) concerning the structures E, F and G is (are)



(E)



(F)



(G)

(A) E, F and G are resonance structures
(B) E, F and E, G are tautomers
(C) F and G are geometrical isomers
(D) F and G are diasteromers

Sol. **(B), (C), (D)**

56. A solution of colourless salt **H** on boiling with excess NaOH produces a non-flammable gas. The gas evolution ceases after sometime. Upon addition of Zn dust to the same solution, the gas evolution restarts. The colourless salt (s) **H** is (are)

(A) NH_4NO_3
(B) NH_4NO_2
(C) NH_4Cl
(D) $(NH_4)_2SO_4$

Sol. **(A, B)**

$$NH_4NO_3 + NaOH \longrightarrow NH_3 + NaNO_3 + H_2O$$
$$7NaOH + NaNO_3 + 4Zn \rightarrow 4Na_2ZnO_2 + NH_3 + 2H_2O$$
$$NH_4NO_2 + NaOH \rightarrow NaNO_2 + NH_3 + H_2O$$
$$3Zn + 5NaOH + NaNO_2 \rightarrow 3Na_2ZnO_2 + NH_3 + H_2O$$

SECTION – III
Reasoning Type

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

---

57. **STATEMENT-1:** For every chemical reaction at equilibrium, standard Gibbs energy of reaction is zero.

and

STATEMENT-2: At constant temperature and pressure, chemical reactions are spontaneous in the direction of decreasing Gibbs energy.

(A) STATEMENT – 1 is True, STATEMENT-2 is True; STATEMENT -2 is correct explanation for STATEMENT-1
(B) STATEMENT – 1 is True, STATEMENT-2 is True; STATEMENT -2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT – 1 is True, STATEMENT-2 is False
(D) STATEMENT – 1 is False, STATEMENT-2 is True

Sol. **(D)**

At equilibrium $\Delta G = 0$, ΔG^{O} of a reaction may or may not be zero.

For a spontaneous process $\Delta G < 0$

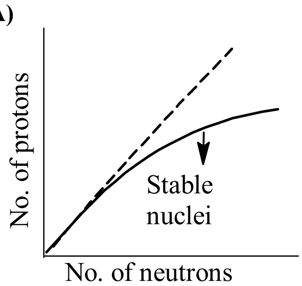
58. **STATEMENT-1:** The plot of atomic number (y-axis) versus number of neutrons (x-axis) for stable nuclei shows a curvature towards x-axis from the line of 45° slope as the atomic number is increased.

and

STATEMENT-2: Proton-proton electrostatic repulsions begin to overcome attractive forces involving protons and neutrons and neutrons in heavier nuclides.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(A)**



If the curve does not bend down towards the x axis then the proton-proton repulsion would overcome the attractive force of proton and neutron. Therefore, the curve bends down.

59. **STATEMENT-1:** Bromobenzene upon reaction with Br_2/Fe gives 1, 4 – dibromobenzene as the major product.

and

STATEMENT-2: In bromobenzene, the inductive effect of the bromo group is more dominant than the mesomeric effect in directing the incoming electrophile.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(C)**
In bromobenzene, it is the mesomeric effect which directs the incoming electrophile.

60. **STATEMENT-1:** Pb^{4+} compounds are stronger oxidizing agents than Sn^{4+} compounds.
and
STATEMENT-2: The higher oxidation states for the group 14 elements are more stable for the heavier members of the group due to 'inert pair effect'.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

Sol. **(C)**
The lower oxidation states for the group 14 elements are more stable for the heavier member of the group due to inert pair effect.

SECTION – IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 61 to 63

There are some deposits of nitrates and phosphates in earth's crust. Nitrates are more soluble in water. Nitrates are difficult to reduce under the laboratory conditions but microbes do it easily. Ammonia forms large number of complexes with transition metal ions. Hybridization easily explains the ease of sigma donation capability of NH_3 and PH_3. Phosphine is a flammable gas and is prepared from white phosphorus.

61. Among the following, the correct statement is
(A) Phosphates have no biological significance in humans
(B) Between nitrates and phosphates, phosphates are less abundant in earth's crust
(C) Between nitrates and phosphates, nitrates are less abundant in earth's crust
(D) Oxidation of nitrates is possible in soil

Sol. **(C)**

62. Among the following, the correct statement is
(A) Between NH_3 and PH_3, NH_3 is better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional
(B) Between NH_3 and PH_3, PH_3 is better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(C) Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional
(D) Between NH_3 and PH_3, PH_3 is better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional

Sol. **(C)**
On going from top to bottom in nitrogen group the bond angle decreases due to more p-character in the bond pair and subsequently more s-character in lone pair orbital.

63. White phosphorus on reaction with NaOH gives PH_3 as one of the products. This is a

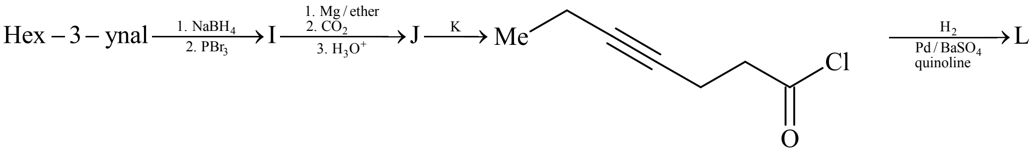
(A) dimerization reaction
(B) disproportionation reaction
(C) condensation reaction
(D) precipitation reaction

Sol. **(B)**

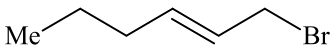
$$P_4 + 3NaOH + 3H_2O \longrightarrow 3NaH_2PO_2 + PH_3$$

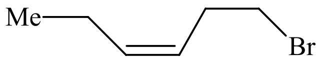
Paragraph for Question Nos. 64 to 66

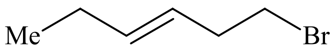
In the following sequence, products I, J and L are formed. K represents a reagent.

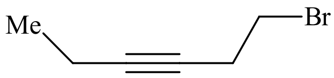


64. The structure of the product I is

(A) 

(B) 

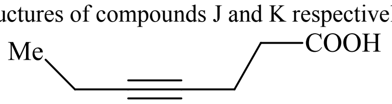
(C) 

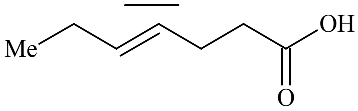
(D) 

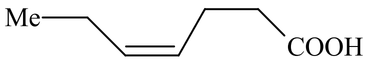
Sol. **(D)**

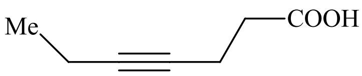
$$CH_3-CH_2-C\equiv C-CH_2-CHO \xrightarrow[\text{2. PBr}_3]{\text{1. NaBH}_4} CH_3-CH_2-C\equiv C-CH_2-CH_2Br$$

65. The structures of compounds J and K respectively are

(A)  and $SOCl_2$

(B)  and SO_2Cl_2

(C)  and $SOCl_2$

(D)  and CH_3SO_2Cl

Sol. **(A)**

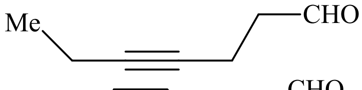
$$CH_3-CH_2-C\equiv C-CH_2-CH_2Br \xrightarrow[\text{3. H}_3\text{O}^+]{\text{1. Mg/Ether; 2. CO}_2} J \xrightarrow{K} CH_3-CH_2-C\equiv C-CH_2-COCl$$

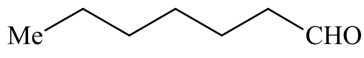
$J = CH_3-CH_2-C\equiv C-CH_2-COOH$

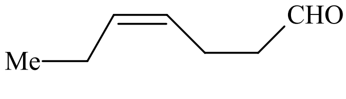
$K = SOCl_2$

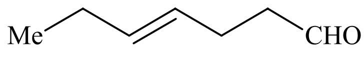
Hence, (A) is the correct answer.

66. The structure of product L is

(A) 

(B) 

(C) 

(D) 

Sol. **(C)**

Paragraph for Question Nos. 67 to 69

Properties such as boiling point, freezing point and vapour pressure of a pure solvent change when solute molecules are added to get homogeneous solution. These are called colligative properties. Applications of colligative properties are very useful in day-to-day life. One of its examples is the use of ethylene glycol and water mixture as anti-freezing liquid in the radiator of automobiles.

A solution M is prepared by mixing ethanol and water. The mole fraction of ethanol in the mixture is 0.9.

Given: Freezing point depression constant of water $\left(K_f^{water}\right) = 1.86\ K\ kg\ mol^{-1}$

Freezing point depression constant of ethanol $\left(K_f^{ethanol}\right) = 2.0\,K\ kg\ mol^{-1}$

Boiling point elevation constant of water $\left(K_b^{water}\right) = 0.52\,K\ kg\ mol^{-1}$

Boiling point elevation constant of ethanol $\left(K_b^{ethanol}\right) = 1.2\,K\ kg\ mol^{-1}$

Standard freezing point of water = 273 K
Standard freezing point of ethanol = 155.7 K
Standard boiling point of water = 373 K
Standard boiling point of ethanol = 351.5 K
Vapour pressure of pure water = 32.8 mm Hg
Vapour pressure of pure ethanol = 40 mm Hg
Molecular weight of water = 18 g mol^{-1}
Molecular weight of ethanol = 46 g mol^{-1}

In answering the following questions, consider the solutions to be ideal dilute solutions and solutes to be non-volatile and non-dissociative.

67. The freezing point of the solution **M** is

(A) 268.7 K
(B) 268.5 K
(C) 234.2 K
(D) 150.9 K

Sol. **(D)**

$$\Delta T_f = K_f \times m$$

$$2 \times \frac{0.1}{0.9 \times 46} \times 1000 = 4.83\ K$$

Freezing point of solution M = 155.7 – 4.83 = 150.87 K $\approx$ 150.9 K

68. The vapour pressure of the solution **M** is

(A) 39.3 mm Hg
(B) 36.0 mm Hg
(C) 29.5 mm Hg
(D) 28.8 mm Hg

Sol. (B)

P = 0.9 × 40 = 36 mm Hg

69. Water is added to the solution **M** such that the fraction of water in the solution becomes 0.9. The boiling point of this solution is

(A) 380.4 K
(B) 376.2 K
(C) 375.5 K
(D) 354.7 K

Sol. **(B)**

$$\Delta T_b = K_b \times m$$

$$= 0.52 \times \frac{0.1}{0.9 \times 18} \times 1000 = 3.2\ K$$

T_b = 373 + 3.2 = 376.2 K